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Self-Interacting Electromagnetic Fields and a Classical Discussion on the Stability of the Electric Charge

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Abstract

The present work proposes a discussion on the self-energy of charged particles in the framework of nonlinear electrodynamics. We seek magnetically stable solutions generated by purely electric charges whose electric and magnetic fields are computed as solutions to the Born-Infeld equations. The approach yields rich internal structures that can be described in terms of the physical fields with explicit analytic solutions. This suggests that the anomalous field probably originates from a magnetic excitation in the vacuum due to the presence of the very intense electric field. In addition, the magnetic contribution has been found to exert a negative pressure on the charge. This, in turn, balances the electric repulsion, in such a way that the self-interaction of the field appears as a simple and natural classical mechanism that is able to account for the stability of the electron charge.

Key-words: Born-Infeld; Charge stability; Negative pressure.

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1   INTRODUCTION

By adopting a nonlinear approach to electrodynamics, in a previus work we have found that an electric charge at rest generates a regular magnetostatic field [1]. The present work investigates how nonlinearity can be used to reveal the presence of an intrinsic angular momentum and to explain the mechanism that holds the electric charge together, ensuring its stability. That finding is very interesting since a classical nonlinear electrodynamics approach to describe the field interaction has naturally led to the electronic spin. In addition a second major result is presented. The calculations have shown that the field interacts with itself creating a negative pressure in the charge that is strong enough to prevent it from bursting. Such finding could be a solution to the historical problem of electron stability. We also point aou that the non-linearity may be the key to the understanding of a number of microscopic effects [2].

It still remains to be found if Maxwell's field equations are to be considered approximations of a more general nonlinear electrodynamical theory. The most physically unpleasant aspect of Coulomb's law is its singularity, that may lead to unbounded field strengths inside charges and thus to an infinite self-energy. Since extremely high electrostatic field strengths are to be found in the vicinity of elementary charges, such regions cannot be accurately described by linear electrodynamics and thus are likely to be associated to departures from Coulomb's law predictions.

This paper is outlined as follows. In the First Section, we briefly describe the Born-Infeld (B-I) Electrodynamics[3][4][5] magnetostatic field solution and we set a correlation with experimental data for the electron. In addition, it includes the calculation of the classical angular momentum due to the intrinsic field and compares it with the value predicted for the quantum spin of the electron. The Second Section describes the calculation of the field pressure produced by the anomalous magnetostatic field. Finally, the Third Section summarizes the main conclusions and our Final Considerations.

2   ANGULAR MOMENTUM FROM FIELD SELF INTERACTION

This section briefly describes the solution to the Born-Infeld equations for a standstill electron as well as the calculation of its intrinsic angular momentum.

According to the standard linear electrodynamics, the presence of a standstill electric charged particle creates an electric field only regardless of its strength. However, according to nonlinear electrodynamics, anomalous effects may also occur due to self-interactions of the fields. The accurate description of high field intensities in the vicinity of an electric charge requires the use of a nonlinear approach. Born-Infeld Electrodynamics has been found to be adequate to describe the fields of a charged particle under such extreme condition.

Considering a static point-like electric charge at the origin, the solution to the first Maxwell equation $\overrightarrow{\nabla} \cdot \overrightarrow{D} = e\delta(\overrightarrow{x})$, with e as the elementary charge, is the electric induction $\overrightarrow{D} = \frac{e}{4\pi r^2}\widehat{r}$, that is singular like the solution from a linear theory. If the magnetostatic sector is allowed to become excited by intense electrostatic fields, the Born-Infeld constitutive relation ensues. Under the assumption that the induced magnetostatic field is always less than the maximum field strength b, the Born-Infeld relationship simplifies, although leaving a residual influence of the electric sector:

$$\overrightarrow{H} = \frac{\overrightarrow{B} - \left(\frac{\overrightarrow{E} \cdot \overrightarrow{B}}{b^2}\right)\overrightarrow{E}}{\sqrt{1 - \frac{\overrightarrow{E}^2 - \overrightarrow{B}^2}{b^2} - \left(\frac{\overrightarrow{E} \cdot \overrightarrow{B}}{b^2}\right)^2}} \overset{|\overrightarrow{B}| \ll b}{\longrightarrow} \frac{\overrightarrow{B}}{\sqrt{1 - \frac{\overrightarrow{E}^2}{b^2}}} = \sqrt{1 + \frac{\overrightarrow{D}^2}{b^2}}\overrightarrow{B}, \tag{1}$$

$$\overrightarrow{B} = \frac{\overrightarrow{H}}{\sqrt{1 + \frac{\overrightarrow{D}^2}{b^2}}}. \tag{2}$$

Also, the Maxwell equations $\overrightarrow{\nabla} \cdot \overrightarrow{B} = 0$ and $\overrightarrow{\nabla} \times \overrightarrow{H} = \overrightarrow{0}$ will complete the set of equations needed to describe the fields. Considering the radial and polar components to be dependent on the radius and the polar angle, the solution for the magnetic field polar component of $\overrightarrow{H}$ can be written as [1]:

$$H^{\theta}(r,\theta) = A\frac{f(x)}{r^3}\sin(\theta). \tag{3}$$

The dimensionless function, f, called "form function", describes the transition from the linear to the nonlinear regime and its asymptotic limit is constant:

$$f(x) = \frac{x^2}{0.3234}\left\{\sqrt{x}P_{1/4}^{1/4}\left(\sqrt{1+x^4}\right) - \kappa x\right\} \xrightarrow{r\to\infty} 1. \tag{4}$$

The function $P_{1/4}^{1/4}(z)$ is the associated Legendre function of first kind and $\kappa(\approx 0.82217)$ is the constant that ensures H^{θ} to vanish when r approaches infinity. Far away from the electric charge, $r \gg r_o$, H^{θ} becomes a genuine magnetic dipole moment field given by:

$$H^{\theta} \xrightarrow{r\to\infty} \frac{A}{r^3}\sin(\theta) \tag{5}$$

Dimensionally, the constant A has units of a magnetic dipole so that equation (5) describes the macroscopic view of an intrinsic magnetic dipole moment for the charge considered. Thus, in order to obtain a more realistic solution, the constant A can be assumed to correspond to the intrinsic electron magnetic dipole moment, that is very close to the value of Bohr's Magneton, $\mu_{Bohr} = 9.27 \times 10^{-24} JT^{-1}$, in the MKS System. Such assumption is needed so that we can bring input to our proposal.

Using the constitutive relation (2), with $B^j \ll b$, the magnetic induction can be set equal to [1]:

$$B^{\theta}(r,\theta) = \frac{H^{\theta}(r,\theta)}{\sqrt{1+\frac{\overrightarrow{D}^2}{b^2}}} = \frac{\mu_{Bohr}}{r^3}\frac{x^2 f(x)}{\sqrt{1+x^4}}\sin(\theta). \tag{6}$$

Once the field structure has been determined, the angular momentum associated with the stationary electric charge can be calculated. It is produced by the interaction between the electric and the magnetic fields, which generates an intrinsic angular momentum given by:

$$\overrightarrow{L} = \int \overrightarrow{x} \times \left(\overrightarrow{D} \times \overrightarrow{B}\right) d^3\overrightarrow{x}. \tag{7}$$

It must be highlighted that both fields inside integral (7) are generated by the point-like electric charge. Thus the magnetostatic induction, $\overrightarrow{B}$, is to be regarded as a product of the nonlinearity only. The simple assumption that the intense electric field caused by the electric charge at rest can excite the magnetostatic sector and yield an intrinsic field angular momentum is completely ruled out in any linear approach.

Back to equation (7), only the axial component will be present due to symmetry considerations and allowing $\overrightarrow{L}$ to be projected on the axial dipole axis, the integral becomes:

$$L_z = \int (r)\left(\frac{e}{4\pi r^2}\right)\left[\mu_o\left(1+\frac{\overrightarrow{D}^2}{b^2}\right)^{-1/2}\frac{\mu_{Bohr}}{r^3}f(x)\sin(\theta)\right]$$

$$\times \sin(\theta)r^2\sin(\theta)d\theta d\varphi dr. \tag{8}$$

This integral can be evaluated and written in compact form for Born-Infeld parameters and natural constants. Defining

$$\gamma = \int \frac{f(x)dx}{\sqrt{1+x^4}} = 1.18,$$

and the B-I radius, recalculated by Born and Schrödinger [7] as:

$$r_o \simeq 2.618 \times 10^{-14} m,$$

then the axial component can be written as:

$$L_z = \frac{2}{3}\left(\frac{\gamma}{r_o}\right)(e\mu_o\mu_{Bohr}) \simeq 0.556 \times 10^{-34} Js. \tag{9}$$

This value for the spin of the electron, obtained on purely classical grounds, departs about 5% only in comparison with the prediction from Quantum Mechanics, $\hbar/2$. It is driven by the nonlinearity. Inside the first parenthesis are the B-I parameters while enclosed in the second one are natural constants. No mechanical rotation or other kind of translation has been considered in order to generate $\overrightarrow{L}$, so that the angular momentum appears naturally, as a remarkable consequence of the self-interaction of the fields. The importance of this result lies not only in its numerical value, but in how the charge produces its intrinsic angular momentum, interpreted here as the spin of the charged particle.

The net result of this section is that the interaction between the electric sector and the magnetic sector generates a spin.

3   THE ELECTRIC CHARGE STABILITY

This section tackles the delicate issue concerning the stability of the electric charge. We present here the details of our claim: the field self-interaction is responsible for the electric charge stability.

The dynamical properties of the electromagnetic field are described by the energy-momentum tensor, $T^{\mu\nu}$. Among its components, T^{rr} is of particular interest because it expresses the radial force per unit area. In terms of Born-Infeld Lagrangian, $\mathcal{L}_{BI}$, T^{ij} [6] is written as:

$$T^{ij} = -E^i D^j - H^i B^j + \delta^{ij} \left\{ \mathcal{L}_{BI} + \overrightarrow{H} \cdot \overrightarrow{B} \right\}. \tag{10}$$

Only the T^{rr}-component is of interest, since the others will not contribute to the radial pressure:

$$T^{rr} = -E^r D^r + \mathcal{L}_{BI} + H^\theta B^\theta. \tag{11}$$

Integrating its projection over axial dipole axis, it becomes, in MKS System:

$$\int_{\substack{hemisphere \\ surface}} (dS\widehat{r}) \, T^{rr} \, (\widehat{r} \cdot \widehat{z}) = \epsilon_o b^2 \pi r_o^2 P(x), \tag{12}$$

where

$$P(x) = -\frac{1}{\sqrt{1+x^4}} + \left(1 - \frac{x^2}{\sqrt{1+x^4}}\right) x^2 + \tag{13}$$
$$+ \frac{1}{2} \left(\frac{4\pi\mu_{Bohr}}{ecr_o}\right)^2 \frac{f^2(x)}{x^2\sqrt{1+x^4}}.$$

The constants, ϵ_o and c, in (12), are the vacuum electric permittivity and speed of light, respectively. The term $\epsilon_o b^2$ is the characteristic field pressure and its module is about $10^{25} N/m^2$. This is a very high pressure. Considering the hemisphere area, $2\pi r_o^2$, where the integration will be performed, the intensity of that particular force is in the order of $10^{-2} N$. In addition, the function $P(x)$ expresses the competition between the outward electrical repulsion and the inward magnetostatic pressures acting on the spherical surphace. The last term, inside $P(x)$, is due to the self interaction of the magnetostatic field. In its absence, the pressure becomes purely repulsive, regardless the sign of the electrical charge. In contrast, its presence promotes a drastic change. All calculations were carefully performed in MKS units system. The balance between forces is depicted in $Figure$ 1 for a hemisphere. It clearly shows the change in sign of the net pressure as well as the drastic changes in its magnitude.

Closing this section the following list summarizes the major nonlinear electrodynamics effects and their causes.

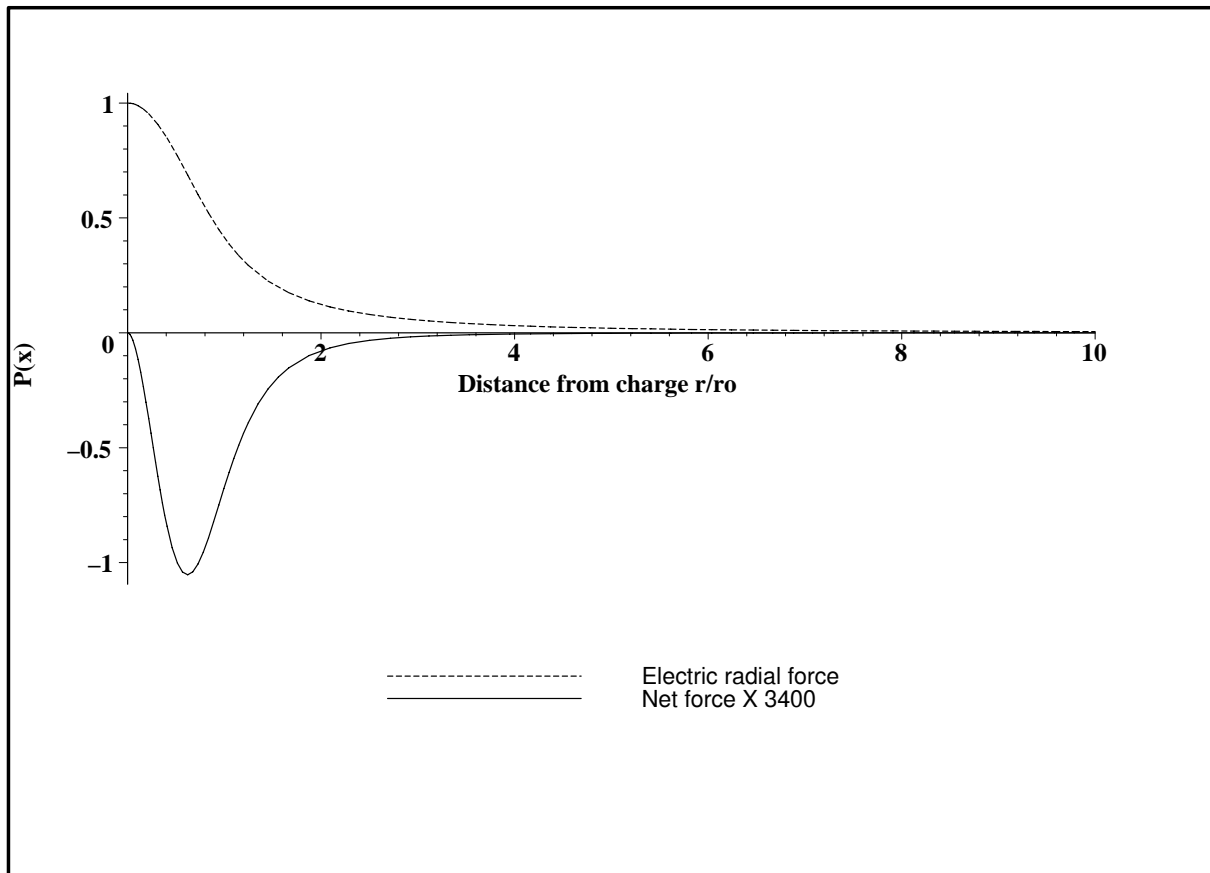| Effect		Cause
Anomalous Magnetostatic Fields	$\Rightarrow$	Extremely High Electrostatic Field
Intrinsic Angular Momentum (Spin)	$\Rightarrow$	Electric and Magnetic Field Interaction
Charge Stability	$\Rightarrow$	Magnetostatic field Self-Interaction

Figure 1

4   FINAL CONSIDERATIONS

The results presented illustrate how accurately non-linearity can represent physical phenomena. However, in spite of the apparent self-consistency of this work, it must be stressed that it would be premature to claim that it actually presents a legitimate description of Nature. The fact that the intrinsic angular momentum of the electron, its spin, could be predicted with a deviation of about 5% only, suggests that predictions for the net pressure are consistent. In other words, the stability of the electronic charge may be described in terms of the self-interaction of the magnetostatic fields. In the present paper, it has been proven that a nonlinear self-interaction mechanism can explain the stability of the charge distribution. Even if it yields some considerable difference with respect to mesurable value, we believe that, qualitatively, it is relevant to understand (classically) how self-interaction and stability are related. This is a lesson we can implement in the framework of Yang-Mills theories.

A further step may be taken at this point. Expression (13) can be rewritten as a function of electronic spin $\overrightarrow{L}$ and thus be differently interpreted. Setting $\frac{\mu_{Bohr}}{r_o} = \frac{3L_z}{2\gamma e \mu_o}$ from (9), and inserting it in (13), yields a connection between the pressure and the spin. Difining $u(x)$ and $v(x)$ as:

$$u(x) = -\frac{1}{\sqrt{1+x^4}} + \left(1 - \frac{x^2}{\sqrt{1+x^4}}\right) x^2,$$

$$v(x) = \left(\frac{6\pi}{e^2 c \mu_o}\right)^2 \frac{f^2(x)}{x^2 \sqrt{1+x^4}},$$

leads to

$$P(x) = u(x) + v(x)L_z^2. \tag{14}$$

It can be easily seen that the term $v(x)L_z^2$ guarantees the stability of the electric charge. It must then be concluded that the presence of spin is necessary to ensure the integrity of the elementary charged particle and that a spinless (truly) elementary charged particle is not expected to exist. This result is in perfect agreement with the fact that no spinless charged (trully elementary) particle has been discovered so far. However, the Minimal Supersymmetric Standard Model (MSSM) predicts the existence of two charged spinless Higgs bosons, in disagreement with the approach proposed here. On the other hand, such particles have not been detected yet and still remain as a theoretical possibility. There is also the possibility that, once they are found (at LHC, for instance), they turn out to be composite structures and not genuinely elementary particles.

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