ISSN 0029-3865

CBPF - CENTRO BRASILEIRO DE PESQUISAS FÍSICAS

Rio de Janeiro

Notas de Física

CBPF-NF-021/08
August 2008

Time And Space Deformations In Special Relativity And Measuring Units

Alfredo Marques

Ministério da Ciência e Tecnologia
BRASIL UM PAÍS DE TODOS GOVERNO FEDERAL

CBPF-NF-021/08

Time And Space Deformations In Special Relativity And Measuring Units*

Alfredo Marques

Centro Brasileiro de Pesquisas Físicas,

Rua Dr. Xavier Sigaud n^0 150,

22290-180, Rio de Janeiro, RJ, Brazil

Abstract

Change of clock rates and the deformation of solid bodies following relativistic transformation between inertial reference systems are discussed under the principles of the physical theory of measurements. Both, time dilation and contraction of rods, are interpreted as being the result of measurements done under different unit standards rather than as a consequence of any physical action born in relative motion. The choice of units is not arbitrary but fixed by metric information contained in a light signal observed under the conditions of the transverse Doppler effect.

*Submitted for publication to American Journal of Physics.

I. INTRODUCTION

Anyone who taught a first course in Special Theory of Relativity has experienced the feeling of incredulity of many students to face the fact that, as a consequence of their relative motion, observers' clocks take up different rhythms and rods become shorter in length. It is true that some gedankenexperiments[1] are quite effective in bringing comprehension to those questions by proving that the effects are expected as a consequence of the fact that time is not the same in different inertial systems; however the argument runs on geometrical rather than physical grounds, leaving unanswered many questions. For instance, it is not clear at all how relative motion acts on different clocks' mechanisms in such a way as to impose the same delay in rhythm, no matter how different they are in working principles - mechanical, electronic, atomic or biological. Unanswered are also the questions raised by the contraction of rods, say, which and where are the applied forces that determine contraction, as well as where is gone the reaction of the rod to the potential elastic energy it stores following contraction.

Since the early days of Special Relativity many authoritative voices raised questions against time dilation and length contraction, claiming that the theory of relativity was unacceptable; as soon as the world of microphysics disclosed movements performed with velocities comparable to the velocity of light propagation in vacuum, experimental observation favoured predictions of Special Relativity in many cases where classical physics failed. Fitzgerald contraction was taken as an experimental fact already at the end of XIX century; in the thirties, cyclotron acceleration was invented and the problems caused by loss of sinchronization between the radiofrequency electric field and the period of motion of the accelerated particles was competently solved by modulating the radio frequency according to the relativistic formula for the variation of angular frequency of the particles: the success of the solution ensued the artificial production of the pi meson.[2] Soon after the discovery of pi-mu decay in 1947, the number of muons at sea level was found incompatible with the measured lifetime and the height of formation in atmosphere: the discrepancy was accounted for as an effect of time dilation of the lifetime of high speed muons, increasing their chances to survive down to the sea level.The measurement was followed by other experiments in accelerators dealing with the lifetime of muons, which favoured time dilation. The outstanding discovery of the transversal Doppler effect[3] following a suggestion by Einstein[4] in 1907, stands as a remarkable proof of time dilation. A revival of Langevin travelers story or, as it became known later on, the twin paradox, was performed by replacing twin brothers by cesium clocks and space ships by commercial air flights, with a result that confirms prediction supported by time dilation[5].

Although some of those observations are rather indirect, that is, do not treat specifically the direct exchange of time and space information between observers in inertial systems moving relatively to each other and either the inertial character of one of the systems is only approximate or, in some other cases, not even that, the list is often considered as an eloquent document in favour of Special Theory of Relativity, good enough to suspend judgement of the pending questions. After the list of successes, questions remained not against the acceptability of the theory but against the abstract treatment that leaves no space to interpret physically time dilation and length contraction.[6]

II. TRANSVERSE DOPPLER SHIFT, MEASURING UNITS, TIME AND SPACE DEFORMATIONS

Transverse Doppler effect is the cleanest proof of time dilation. It foolows from the fundamentals of Special Theory of Relativity that the frequency, ν, of light radiation emitted by moving sources and observed perpendicularly to the direction of movement, is related to the frequency, ν_0, measured by an observer at rest with respect to the source, by

$$\nu = \nu_0 \gamma \tag{1}$$

where:

$$\gamma = \sqrt{1 - V^2/c^2} \tag{2}$$

and V the relative velocity. From $\gamma = \nu/\nu_0 = T_0/T$, time dilation of the oscillation period, T, follows at once.

Transverse Doppler shift also brings information on a sort of natural unit quite relevant to the main conclusions of this text. Before discussing this point, however, it is useful to recall the basic relationship connecting the physical quantities under measurement, the result of their measurement (or their measure) and the measuring units applied. Let q be any physical quantity, U its measuring unit and χ the result of its measurement or its measure:

$$\chi \times [U] = [q] \tag{3}$$

Square brackets are meant to emphasize the dimensional character of the parameters involved; χ, a non dimensional integer, stands for the number of times the measuring unit U has to be successively applied to match the quantity q. When no integral number of applications of the measuring unit is enough, other integer numbers in lower orders of significance may appear, representing the repeated application of sub-multiple units, until the match is considered satisfactory. Therefore, in general, χ will be a non dimensional real number.

Whenever two observers are interested in doing independent measurements on common physical objects they have, first of all, to agree upon measuring units. Following the prescriptions of international unit systems usually does the job, but very often secondary standards available to both observers are also required for the sake of improved precision.

The frequency ν, of a periodic motion, in s^{-1} is the number of complete oscillations performed in 1 second; the same number represents the non dimensional measure of the time interval of 1 s in units of the period of oscillation, T, in seconds: $\nu = 1/T$. Suppose the observer in S_0, sends to the observer in S, moving with velocity V relatively to S_0, a sample of the measured value of the interval of 1 s, materialized in the frequency, ν_0 of a monochromatic light sign emitted by a source at rest in S_0. Suppose the light signal is detected by the observer in S at right angle with the direction of the relative velocity, V. After the spectral analysis of the signal, it is correct to say, following what is known about transverse Doppler effect, that the interval of 1 second has now a shorter duration, since it intersects a smaller number of frequency oscillations: the shortening factor is γ. Now suppose a time interval of duration Δt is measured by observers, one in S_0, other in S; let τ_0 and τ be the measures of Δt in the measuring units holding in systems S_0 and S, respectively. Then, according to (3), one has:

$$\tau \times \gamma = \tau_0 \times 1 = [\Delta t] \ , \tag{4}$$

since the time unit in system S_0 is one second and, in system S, (one second)$\times\gamma$. From the relation above:

$$\tau = \tau_0/\gamma \tag{5}$$

Time dilation, therefore, appears as the result of changing measuring units rather then as a consequence of inertial relative motion. It is worth to comment that the time interval $[\Delta t]$ is the same for both observers, only its measure is different. Relativity of time is then the relativity of its measures.

The observer in S must use a clock appropriated to the measuring unit. He has two options: either to change the angular scale in the dial by shortening by a factor γ the unit angular interval displayed, or to leave the display scale untouched but acting on the "fast-slow" device available so as to lower clock's pace to the appropriate value. The usual interpretation of time dilation represents a third option: doing nothing, i.e. using a clock identical to the one in S_0. In that case he will be led to the conclusion that his clock became slower.

Light propagation velocity, c, when expressed in same units for length and time, must be given by the same number for both observers. Let the observer in S_0 express it in m/s. Since the time unit in S is the second shortened by the factor γ, any time intervals measured in units of that system will be converted into seconds by dividing it by that factor. Therefore, the unit of length in S will be converted into meter by division by the same factor so as to obtain the same value for c. That procedure governs the conversion into meter of the measure of any straight length taken in the measuring unit holding for S: if λ_0 and λ are measures in S_0 and S of the same length $[l]$ one has:

$$\lambda = \lambda_0 \gamma \tag{6}$$

Therefore, Fitzgerald contraction appears as an effect accompanying the diferent units of length in inertial systems with relative velocity V. Also in this case, length interval is the same for both observers, only its measures are different depending on the system where they are performed.

As a matter of fact it is not strictly required for the radiation detected in system S to be at right angles to the direction of relative velocity. Relativistic Doppler shift is present in any direction; as the angle of observation goes closer to the direction of relative motion the effect is masked by the radial Doppler shift. Radial Doppler shift is more intense than the transverse one by terms of the order V/c, and reflects the same circumstances governing non relativistic Doppler shift. Detecting the radiation at large angle with the direction of relative motion is not, therefore, a matter of principle but a consequence of the present limits of the experimental art in separating the radial component from the transversal one. Relativistic Doppler shift, with its bearings on measuring units of length and time in inertial systems moving relatively, is a consequence of the main assumption of Special Relativity that light propagation velocity is the same in any direction, no matter the speed of the emitting source.

It is also worth to say that the coordinate axes of time and position along the direction of relative motion in systems S_0 and S must be headed by a set of measuring units such as described above. In those units the coordinates will be such numbers that the wave fronts will appear to be spherical to the observer in S_0 as well as to the observer in S. The expression of constant velocity and isotropic propagation of light is condensed in Lorentz transformations, written in those coordinates.

The interpretation above will presumably save a lot of trouble to overcome the feeling of rejection that the standard interpretation of time dilation and length contraction raises in the mind of students, as well as to shorten the time spent by teatchers to deal with all associated difficullties while retaining objectivity of natural science.

The author is indebted to F. Caruso, S.B.Duarte, T. Kodama, A. Macedo, M. Novello, A.F.S.Santoro and A.F.F.Teixeira for kindly reading and criticizing the manuscript.

---

[1] W.F.Panofsky and M. Phillips, *Classical Electricity and Magnetism,* (Addison-Wesley, Mass. 1962, pg 289-291), 2nd. ed.

[2] E. Gardner and C.M.G.Lattes, *Production of mesons by the 184- inch Berkeley cyclotron,* (Science, 107, 1948, 270-271).

[3] H.E.Ives, G.R. Stilwell, *An Experimental Study of the rate of a Moving Atomic Clock,* (J.Opt.Soc. Am. 28, 215 1938).

[4] as quoted by A. Harvey and E. Schucking, *A Small Puzzle from 1905,* (Physics Today, March 2005, pg.34).

[5] J.C.Haffele and R.E. Keating, *Around the World Atomic Clocks: Predicted Relativistic Time Gains,* (Science. 177, 166 1972); Around the World Atomic Clocks: Observed Relativistic Time Gains (Science 177, 167 1972).

[6] Remarkable are the efforts of J.S.Bell in the chapter on how to teach special relativity in *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, 1987 and L. Jánossy *Über die physikalische Interpretation der Lorentz-Transformation*, (Ann. der Phys. 6, 11, 1953).