

# Do confinement and darkness have the same conceptual roots?

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## Abstract

Indecomposable positive energy quantum matter comes in 3 forms: one massive and two massless families of which about the so called "infinite spin" family was little known up to recently.

Using novel methods which are particularly suited for problems of localization, it was shown that this quantum matter of the third kind cannot be generated by pointlike localized fields but rather needs semiinfinite stringlike generators. Arguing that the field algebras generated by these new objects do not possess any compactly localizable subalgebras, we are led to a situation of purely gravitating matter which cannot be registered in any particle counter i.e. to observational darkness and possibly also inertness. A milder form of darkness which only blackouts certain string localized objects but leaves a large observable subalgebra generated by pointlike fields occurs with interacting zero mass finite helicity matter and it is the main aim of this note to emphasize these analogies.

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# 1 Positive energy matter as classified by Wigner and invisibility of particles

Partially invisible quantum matter in the sense of this note is quantum matter which has no charge coupling to photons but whose weak interaction with visible quantum matter may still permit an indirect counter registration as the various proposals for the astrophysical dark matter in the form of WIMP.

A more extreme case, to which we want to direct the reader's attention in the sequel, is *completely dark matter*. As all positive energy matter, this quantum matter has a gravitational manifestation, but it permits no compact localization and consequently cannot be registered in laboratory counters. Although such objects did not yet enter the particle physics scene in connection with the present hunt for the physical identification of dark matter, they existed in a concealed not understood form ever since Wigner in 1939 wrote his famous paper on unitary irreducible ray-representations [1] of the Poincaré group. As will be argued in this note, with the recent unravelling of some of their properties these representations represent "darkness" in its most extreme occurrence and to propose them as candidates for dark matter is irresistible. A shorter account has been given in form of a letter [2].

Wigner found that there are precisely *three families of indecomposable positive energy representations*. They are distinguished by the nature of the little group and its representation theory. Besides the best studied massive representations which constitute the first kind for which the little group  $SO(3)$  is the invariance group of a timelike vector, there exist two massless families whose little group leaves a lightlike vector invariant and is isomorphic to the noncompact euclidean group, which is a subgroup of the Lorentz group  $E(2) \subset L(3,1)$ . The Casimir invariant of the  $E(2)$  representation is a kind of continuous "euclidean mass" and since the representation of the P-group is induced from  $E(2)$ , this property is passed on to the  $P$ -representation. As we will see, the ability for compact spacetime localization decreases when one passes from the first kind of massive matter via the second massless finite helicity matter to the third kind [15]. We will show that this results in an increase of invisibility starting from the fully visible massive matter passing through partial invisibility (gluonic confinement) up to total darkness (inertness) for the third kind of Wigner quantum matter.

What distinguishes the two massless families is the *nature of the  $E(2)$  representations*; whereas the finite helicity family, which contains the known zero mass particles is a degenerate (not faithful) representation in which the euclidean translation is represented trivially (which compactifies the representation despite the noncompactness of the group), the third family results from a faithful  $E(2)$  representation which preserves the group theoretic noncompactness on the level of the representations and comes with unusual and conceptually demanding properties. The "little Hilbert space" is now an infinite dimensional space of Fourier components which describe an  $E(2)$ -irreducible infinite intrinsic abelian angular momentum tower; this is why we prefer to use "infinite spin" over Wigner's "continuous spin" (which refers to the continuous values of the Casimir invariant).

The appearance of this infinite spin tower prevents the extension of the P-group to the conformal group despite the vanishing of the mass, like the massive family it is not conformally invariant and both the massive representation and the third family have a

continuous Casimir cardinality (which in the degenerate massless family is only countably discrete). In fact none of the standard attributes of masslessness (e.g. the Huygens principle) hold for these infinite spin tower matter; *in particular those arguments which led to the exclusion of light neutrinos as WIMPs are not applicable with respect to massless third kind quantum matter.*

Only recently [3] it became clear why the more than 60 year struggle to understand the quantum field theoretic content of this huge family of indecomposable (particle-like) positive energy representations resisted all attempt of incorporation into a Lagrangian quantization setting. It turned out that this third kind of matter is generated by noncompact extended singular objects which are spacelike semiinfinite covariant string-like-localized fields<sup>1</sup>.

Already Wigner was fascinated by these extreme quantum objects for which apparently his intrinsic (independent of any quantization) representation-theoretical setting was the *only* access since any subsequent attempt to understand them in terms of "quantization" in the sense of a classical-quantum parallelism led him nowhere. When he found out in 1948 [4] that there were apparent problems with placing such objects into a thermal state<sup>2</sup>, he begun to have doubts about their physical utility. The subsequent investigation of localization properties which unfortunately consisted in trying to press this family into the standard quantization scheme for pointlike covariant fields instead of following its own rules was pursued by several generations of particle physicists and ended in inconclusive results [5].

The most laconic way to exorcise this apparent conceptual nuisance of the presence of third kind of matter can be found in Weinberg's 1995 excellent first volume [6]. In contrast to most other textbooks he does present these representations but then dismisses them with the remark that nature has apparently no use for them; this leaves the reader without a clue if any principle of nature was possibly violated by this matter.

If one wants to argue whether nature could realize these representations in a possibly more discrete form which hitherto escaped our detection, one would have to know their conceptual status much better than this was the case at the writing of Weinberg's book. We know nowadays, that with those conceptual instruments available at that time, he could not have gone further.

Actually there was an unheeded early hint towards a new direction in a 1970 paper [8] in which a mathematically precise no-go theorem was derived, proving that the infinite spin representation cannot be obtained within the setting of covariant pointlike local free fields (the Wightman framework). But only by the end of the 90, when the conceptual-mathematical tools were in place, a good part of their physical properties, in particular about their precise localization status, begun to unravel.

It will be shown in the sequel that localization of the third kind Wigner matter is noncompact; i.e. a compact spacetime region is associated with a trivial subspace of Wigner wave function (zero vector) and the associated field algebra is a multiple of the identity. The best localized generators of that representation are spacelike semiinfinite covariant stringlike localized<sup>3</sup>. When we recently discovered these unusual properties of

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<sup>1</sup>String localization in the sense of this note is an intrinsic quantum field theoretic concept which is *not valid for the objects of string theory* (see later comments).

<sup>2</sup>These difficulties are explained in terms of the semiinfinite string localization. Such objects cannot be enclosed into a quantization box; i.e. they do not have Gibbs states but do admit KMS states [3].

<sup>3</sup>In an analogous way as a point can be viewed as the limiting case of a simply connected convex

the third Wigner representation family [15] we also obtained a new vantage point for looking at the role of "potentials" associated with the "field strengths" of the zero mass finite helicity ( $m=0,s$ ) representations of which the lowest nontrivial possibility is the  $s = 1$  vectorpotentials. They do exist in the Hilbert space of field strength but only if one permits semiinfinite stringlike localization. Different from the mentioned third kind of quantum matter in the absence of interactions *they do not change the pointlike localizable particle content* (which is fully described by the pointlike field strengths). In the coupling to scalar and spin= $1/2$  matter they play an important role in a better understanding of the infrared problem and the delocalization of charged objects.

A more *radical change from string-like "gluons" comes about if one looks at self-coupling* of such stringlike objects and demands that there exists a pointlike generated subalgebra. It turns out that this results in a restriction which is completely equivalent to that obtained from gauge invariance in a gauge theoretic formulation<sup>4</sup>. However now the gluons *are objects in the physical Hilbert space*<sup>5</sup> and the reason why they are not observable is explained in terms of their semiinfinite string localization and not on their ghostly nature. Whereas the representations of the third kind, as will be argued in the sequel, amounts to total darkness (no counter-registration), the second kind of finite helicity massless quantum matter comes with a milder form of confinement as "gluonic" darkness<sup>6</sup>. This spacetime explanation of confinement in terms of partial darkness brings heaven (astrophysics) closer to earthly QCD (LHC)<sup>7</sup> but unfortunately also dims the hope to see DM (which is presumably totally inert apart from gravitation) in laboratory experiments.

The mathematical framework of the relevant quantum localization concept is fairly new (but not revolutionary in the sense of the present use of this terminology in particle physics) and goes under the name of *modular localization* [9][10]. Since in the deafening noise of present particle physics fashions probably none of the readers has taken notice about significant conceptual progress in QFT, I will at least sketch the main idea without

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compact spacetime region, the semiinfinite string is the singular idealization of a semiinfinite noncompact region with corresponding connectivity properties.

<sup>4</sup>The requirement that a coupling of gluons leads to a nontrivial compactly localizable subalgebra of observables has the same restrictive power as gauge invariance in a pointlike setting involving ghosts. But whereas gauge invariance was an important classical selection principle between different couplings involving vector fields and was of invaluable help for finding once way to renormalization, there is no need for such a principle local quantum physics because the requirement that there exists a compactly localized subalgebra in the presence of massless higher spin objects in an arena which is a (ghostfree) ambient Hilbert space. The gauge principle is a technical trick to throw out all string-localized "dark" objects (gluons) in favor of local observables. It has no permanent place in local quantum physics where causal localization is the overriding principle.

<sup>5</sup>As e.g. spinor fields the string localized vector potentials live in the physical Hilbert space but are not observables; in the first case the reason is the behavior under rotations and in the second case it is the fact that every measurement is local or at least quasilocal. In both cases they are extremely useful objects.

<sup>6</sup>More detailed future investigations may reveal that quark confinement is explained in terms of partial darkness. For this one would have to show that, different from QED where the presence of the string potential only delocalizes charged particles into quasilocal objects, the QCD interaction in the string-localized setting causes a more violent delocalization of quarks which is responsible for its darkness (alias confinement).

<sup>7</sup>The help from heaven according to the new ideas outlined in this note is a bit different than in most articles on this subject: astrophysics as a midwife for a radical revision of gauge theory and the standard model?

proof in the simplest spinless case (where also traditional methods would be sufficient) and only quote the results for the case at hand.

Intuitively modular localization results from the causal localization, which is inherent in relativistic QFT, after one liberates it from the use of particular field coordinatizations<sup>8</sup>, i.e. the localization in the standard formulation of QFT is a special case of modular localization. Starting from a Wigner representation space of wave functions of a scalar particle

$$H_{Wig} = \left\{ \psi(p) \mid \int |\psi(p)|^2 d\mu(p) < \infty \right\}, \quad (\mathbf{u}(\Lambda, a)\psi)(p) = e^{ipa}\psi(\Lambda^{-1}p) \quad (1)$$

$$(\mathbf{u}(j_{W_0})\psi)(p) = \overline{\psi(-j_{W_0}p)}, \quad \mathbf{u}(j_{W_0})\mathbf{u}(\Lambda_{W_0})\mathbf{u}(j_{W_0})^* = \mathbf{u}(\Lambda_{W_0}) \equiv \mathbf{u}_{W_0}(\chi) \quad (2)$$

one first defines two commuting operators which are associated to the  $t - x$  wedge  $W_0 = \{x \mid x_1 > |x_0|\}$ : the unitary representers  $\mathbf{u}$  of the wedge-preserving Lorentz boost  $\Lambda_{W_0}(\chi)$  and the antiunitary representer of the wedge-reversing reflection  $j_{W_0}$  across the edge of the wedge (second line). One then forms the<sup>9</sup> “analytic continuation” in the rapidity  $\mathbf{u}(\chi \rightarrow -i\pi)$  which leads to unbounded positive operators.. Using a notation which is customary in modular theory [11], we define the following unbounded closed anti-linear involutive operators in  $H_{Wig}$

$$\begin{aligned} \mathfrak{s}(W_0) &:= j_{W_0}\delta_{W_0}^{\frac{1}{2}}, \quad \delta_{W_0}^{it} := \mathbf{u}_{W_0}(\chi = -2\pi t), \quad \mathfrak{s}^2(W_0) \subset \mathbf{1} \\ &\curvearrowright (\mathfrak{s}(W_0)\psi)(p) = \psi(-p)^*, \quad \text{dom } \mathfrak{s}(W_0) = \text{dom } \delta_{W_0}^{\frac{1}{2}} \end{aligned} \quad (3)$$

where the analytic properties of the domain of this modular involution  $\mathfrak{s}(W_0)$  consists precisely of that subspace of Wigner wave functions which permit that analytic continuation on the complex mass shell which is necessary in order to get from the forward to the backward mass shell ( $\chi \rightarrow \chi - \pi i$ ). The main assertion of modular localization is that the  $\pm 1$  eigenspaces (real since  $\mathfrak{s}(W_0)$  is antiunitary) are the real closed component of the dense  $\text{dom } \mathfrak{s}(W_0)$

$$\begin{aligned} K(W_0) &= \{\psi \mid \mathfrak{s}(W_0)\psi = \psi\}, \quad \mathfrak{s}(W_0)i\psi = -i\psi \\ \text{dom } \mathfrak{s}(W_0) &= K(W_0) + iK(W_0), \quad \mathfrak{s}(W_0)(\psi + i\varphi) = \psi - i\varphi \end{aligned} \quad (4)$$

The dense subspace  $\text{dom } \mathfrak{s}(W_0)$  ( $\overline{\text{dom } \mathfrak{s}(W_0)} = H_{Wig}$ ) is precisely the one-particle component of the  $W_0$  localization space associated with a scalar free field  $A(x)$ , or in terms of the real subspace<sup>10</sup>

$$K(W_0) = \text{clos} \{(A(f) + A(f)^*)\Omega \mid \text{sup } pf \subset W_0\} \quad (5)$$

but the modular construction of localized subspaces avoids the use of singular field coordinatizations smeared with classically localized test functions and relies instead on the

<sup>8</sup>A free field and each of its infinite set of composites generate the same modular localization because the latter only depends on the structure of the generated localized operator algebras and not on properties which distinguish their individual operators.

<sup>9</sup>The unboundedness of the  $\mathfrak{s}$  involution is of crucial importance for the encoding of geometry into domain properties of unbounded operators.

<sup>10</sup>The closedness of  $K$  does not lead to that of  $K + iK$ .

more intrinsic quantum description in terms of domains of distinguished unbounded operators in the unique<sup>11</sup> Wigner space associated with the representation  $(m, s = 0)$ . The second line is the defining relation of what mathematicians call a *standard real subspace*. The standardness property is equivalent to the existence of an abstract (nongeometric) modular involution.

Applying Poincaré transformations one generates from  $\mathfrak{s}(W_0)$  and  $K(W_0)$  to the  $W$ -indexed families  $\{\mathfrak{s}(W)\}_{W \in \mathcal{W}}$ ,  $\{K(W)\}_{W \in \mathcal{W}}$ . The localization spaces for smaller causally complete spacetime regions  $\mathcal{O}$  (which could be trivial) are obtained by intersections  $K(\mathcal{O}) = \cap_{W \supset \mathcal{O}} K(W)$ . A remarkable property of all these spaces resulting from Wigner's positive energy representation setting is the validity of *Haag duality*

$$K(\mathcal{O}') = K(\mathcal{O})' \quad (6)$$

where the dash on the region denotes the causal complement and that on the  $K$ -space stands for its symplectic complement within  $H_{Wig}$  i. e.  $Im(K, \varphi) = 0$  for all  $\varphi \in K(\mathcal{O})' = j_{\mathcal{O}}K(\mathcal{O})$

The final step is the functorial ascend to the net of spacetime localized operator algebras in the Wigner-Fock space (with creation/annihilation operators  $a^*(p), a(p)$ )

$$\begin{aligned} Weyl(\psi) &= \exp i(a(\psi) + a^*(\psi)), \quad \psi \in K(\mathcal{O}) \\ \mathcal{A}(\mathcal{O}) &:= alg \{Weyl(\psi) \mid \psi \in K(\mathcal{O})\}, \quad \mathcal{A} := \cup_{\mathcal{O}} \mathcal{A}(\mathcal{O}) \end{aligned}$$

where *alg* denotes the operator (von Neumann) algebra generated by the unitary Weyl operators in the Wigner-Fock space. Note that there are no spacetime dependent field coordinates, the construction is as intrinsic and unique as the Wigner representation theory.

This modular construction exists for all three Wigner representation families. The  $K(\mathcal{O}) + iK(\mathcal{O})$  spaces for  $\mathcal{O} = \mathcal{D} =$  double cone (the prototype of a simply connected causally complete compact region) for the first 2 families are dense in  $H_{Wig}$  whereas the third kind of Wigner matter yields a vanishing  $K(\mathcal{D})$ . In that case the nontrivial space with the tightest localization  $K(\mathcal{C})$  is associated with an (arbitrarily thin) noncompact spacelike cone  $\mathcal{C} = x + \mathbb{R}_+ \mathcal{D}$  with apex  $x$  and an opening angle which is determined by  $\mathcal{D}$ .

There is no problem in adapting the modular setting to the presence of interactions; however there are no one-particle creators in compactly localized algebras. In order to recover particle creation operators creating one-particle states by acting once on the vacuum, one must go to noncompact regions of the size of a wedge. In that case one can show that there are wedge-localized operators which applied to the vacuum create vacuum polarization-free one-particle states (PFGs [14]). This interesting relation of wedge localization with one-particle states and, as more detailed studies show, the scattering matrix attributes to the latter a completely new role of a relative modular invariant which it did not have in scattering theory. It opens the possibility of modular-based QFT model constructions starting in the first step with the construction of wedge algebras. Such a

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<sup>11</sup>It was precisely this uniqueness which was Wigner's main motivation for bypassing the confusing plurality of the quantization setting (many different equations of motion have the same physical content) in favor of an intrinsic description. However the adaptation of the Born particle localization (the Newton-Wigner localization) unfortunately got him onto the wrong track as far as the causal localization is concerned.

program has been initiated in [9]; it was shown subsequently in [12] that modular theory together with a phase space properties does indeed secure the existence of factorizing models. These modular ideas also lead to some clarification about a precise conceptual meaning of *P-covariant "noncommutative QFT"* [13].

For more realistic higher dimensions on still depends on the perturbation theory of coupled singular field coordinatizations. All the steps explained above in the spinless context can be carried out for the first two families with the help of intertwiners. The use of modular localization theory is not essential, the intertwiners can also be constructed by standard group theoretical techniques without referring to localization as explained in Weinberg's first volume<sup>12</sup> of [6]. In that case they are are intertwiners from the unique Wigner representation to the denumerable set of spinorial representations  $(A, \dot{B})$  whose undotted/dotted indices run over  $(2A + 1)(2\dot{B} + 1)$  indices (from  $-A$  to  $+A$  and  $-\dot{B}$  to  $+\dot{B}$ ) and lead to the following spinorial fields (tensors are a special case in this spinorial formalism)

$$\Phi_r(x) = \sum_{k=-s}^s \int d\mu(p) \{e^{ipx} u_{k,r}(p) a^*(p, r) + e^{-ipx} u_c(p)_{k,r} b(p, r)\} \quad (7)$$

$$m > 0 : \left| A - \dot{B} \right| \leq s \leq A + \dot{B}$$

with an explicit formula for the  $(2A + 1)(2\dot{B} + 1)$ -component interwiner  $u_{k,r}(p)$  and its charge conjugate. Whereas in the massive case a given  $s$  can be alternatively described (local equivalence  $\rightarrow$  physical equivalence with same creation/annihilation operators) by all different pairs  $(A, \dot{B})$  which are only subject to the above inequality, the massless case has gaps in that half of the possibilities admitted by the above inequality are missing as a consequence of the degenerate little group representation. In particular (as noticed by Weinberg [6]) there is no covariant vector potential for  $h = 1$  (and no symmetric tensor in the Hilbert space of the  $h = 2$  "graviton"). On the other hand a covariant semiinfinite string-localized vector potential  $A_\mu(x, e)$  poses no problems i.e. the gaps in the spinorial formalism (7) can be filled with string-localized field generators. These covariant fields, which have a natural construction in the modular localization setting, possess the useful property of a very good short distance property (scale dimension one independent of helicity). They certainly are more intrinsic objects within the Wigner setting extended by modular localization than the contrived ghost extension of the Wigner formalism which tries to maintain the formal pointlike property (and hence the relation to the classical gauge formalism) at any cost.

The perturbative results for the observables, alias gauge-independent composites and now pointlike generators of *compactly generated subalgebras*, are of course expected to coalesce since the new setting is not modifying any gauge invariant result of the old gauge theory setting. But the modular setting is a more intrinsic description of massless vector potentials than the ghostly gauge formalism. The modular formalism maintains the initial Hilbert space throughout the computation as opposed to the change caused by the BRST cohomological descend from an explicitly indefinite metric Fock space to an only implicit constructed physical Hilbert space. As far as localization properties are concerned the

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<sup>12</sup>The modular method explains why the group theoretical principle of covariance and the principle of causal localization are closely related [10].

second kind of Wigner matter is somewhere between the first (most perfectly pointlike) and the third (extreme stringlike as shown in the sequel).

There is no need for stringlike objects in the massive Wigner representation setting. However for *interacting massive  $s \geq 1$  fields the use of stringlike fields* which have the same short distance dimensions as a scalar field independent of spin *is a fascinating alternative* to gauge theory including the Higgs mechanism for maintaining renormalizable interactions involving massive higher spin particles (see last section). The reason for the improved short distance behavior of string like localization is that a string localized field  $A(x, e)$  fluctuates not only in the  $x$  of Minkowski space but also in the spacelike directional unit vector  $e$  which is a point in a 3-dim. de Sitter space (i.e.  $e$  is not a gauge parameter but a fluctuating quantum variable). As a result the scale dimension in  $x$  is lowered from 2 to 1, because part of the fluctuations go with the point  $e$  in de Sitter space [15]. Massive string fields do not show up in the particle spectrum and the scattering theory, the only purpose for their introduction is to enlarge the realm of renormalizable interactions i.e. to encounter more finite parameter (perturbative) QFT without having to resort to BRST ghost methods especially in case of  $s \geq 1$ . There can be no doubt that these massive string localized fields are identical with the singular generators of spacelike cone localized algebras which Buchholz and Fredenhagen found a long time ago in their structural analysis of the relation between spectral properties and the best possible localization [7].

For the third kind of matter the only systematic construction is one which determines a continuous  $\alpha$ -dependent family of intertwiners  $u^\alpha(p, e)$  using their modular localization properties [3][15]. In this way one obtains a continuous set of localizing intertwiners  $u^\alpha(p, e)$  which depend in addition to the momentum  $p$  on a spacelike unit vector  $e$ ,  $e^2 = -1$ . It intertwines the Wigner transformation which involves the representation of the noncompact little group  $D_\kappa(R(\Lambda, p))$  with the covariance transformation law in  $p$  and  $e$  and leads to a string field whose intrinsic stringlike extension can be seen by the appearance of a nontrivial commutator if one string gets into the causal influence region of the other

$$D_\kappa(R(\Lambda, p))u^\alpha(\Lambda^{-1}p, e) = u^\alpha(p, \Lambda e), \quad (8)$$

$$\Psi(x, e) = \left(\frac{1}{2\pi}\right)^{\frac{3}{2}} \int_{\partial V_+} d\mu(p) (e^{ipx} u^\alpha(p, e) \circ a^*(p) + e^{-ipx} \overline{u^\alpha(p, e)} \circ a(p))$$

$$[\Psi(x, e), \Psi(x', e')] = 0 \text{ only for } x + \mathbb{R}_+e \succ\prec x' + \mathbb{R}'e'$$

That certain objects do not admit a presentation in terms of pointlike fields is not a speciality of these infinite spin representation. In  $d=1+2$  "plektons" (particle associated to braid group statistics) are particles whose field theoretic description requires spacelike strings [16]. However by forming bilinear composites one descends to compactly localizable observables. Another case is that of vector fields in zero mass  $h = 1$  representation mentioned before. That there are no compactly localized subalgebras representing observables for the third kind of matter is the main point of the following consideration.

The infinite spin family is string-like in a very radical sense. There is neither a compactly localizable subspace of the Wigner space as in the vectorpotential  $A_\mu(x, e)$ - field strength relation, nor are there composite fields which are local with respect to  $\Psi(x, e)$ . The first statement was proven in [8] and the absence of pointlike localized composites can be supported by the following calculation. The most general covariant bilinear scalar



object in the Wigner infinite spin creation/annihilation operators is of the form [15]

$$\begin{aligned}
B(x) &= \int \int_{\partial V} d\nu(k)d\nu(l)d\mu(p)d\mu(q)e^{i(p+q)x}u_2(p, q)(k, l)a^*(p, k)a^*(q, l) \quad (9) \\
u_2(p, q)(k, l) &= \int d^2z d^2w e^{i(kz+lw)}F(B_p\zeta(z) \cdot B_q\zeta(w)) \\
\zeta(z) &= \left(\frac{1}{2}(z^2 + 1), z_1, z_2, \frac{1}{2}(z^2 - 1)\right)
\end{aligned}$$

where  $F$  is any smooth sufficiently decreasing function so that  $u_2$  is square integrable in  $k, l$  for fixed  $p, q$ . This function is so constructed that  $u_2$  absorbs the complicated Wigner transformations (involving the little group with  $\Lambda$ -dependent parameters) and the net result is a scalar field. The momentum integration is over both light cones  $\partial V = \partial V_+ \cup \partial V_-$  and we use the notation  $a^*(-p) \equiv a(p)$ . According to the Kallen-Lehmann representation its two-point function is automatically causal, but this only means that the distribution-valued vector  $B(x)\Omega$  is point-localized and does not imply the locality of the operator itself. The string generated algebra has local subalgebras in case of existence of tensor fields which are relatively local to the string. A scalar bilinear field as the above  $B$  is a special case for which the impossibility of relative locality is easily shown. The negative answer to the question:

$$\exists B \text{ s.t. } \langle q, l | [B(x), \Psi(y, e)] | 0 \rangle = 0, \quad x \gg y + \mathbb{R}_+ e ? \quad (10)$$

is best understood by comparing the contraction functions with those for standard matter. By splitting off a plane wave exponential the matrixelement in (10) only depends on the difference. The Fourier transform of this function is polynomial in the Fourier momentum and this leads to the spacelike vanishing. The presence of the  $z, w$  little-group Fourier transforms in (9) as well as in the definition of  $\Psi(x, e)$  leads to a much more complicated non-polynomial momentum space dependence which after Fourier transform to the relative distance variable  $x - y$  has no support properties at all. A more pedestrian way to see this is to place the string direction  $e$  into the equal time plane and show that the expected delta function (or its derivative) which equates  $\vec{x}$  with the coordinate on the string  $\vec{y} + \mathbb{R}_+ \vec{e}$  cannot arise.

This situation cannot be improved by going from bilinear scalars to tensors, or by generalizing from bilinear to  $2n$ -linear expressions in the  $a^\#$ . The best one can do is forming composite local strings which at least maintain the original string localization. But the possibilities for constructing string composites is much larger than that given by Wick-polynomials which are the quantum analogs of classical local field functions. Similar to the discussion of relative local fields with respect to so-called generalized free fields [17], there is no classical description of composedness (there are continuously many relative local fields beyond the "classical" Wick polynomials), which again testifies to the *intrinsic quantum nature* of the infinite spin matter.

This may be the right moment to make a clarifying (perhaps already long expected) remark about the relation of string-localized third kind of Wigner matter and the objects of string theory. The "string" in string theory is a metaphorical terminology which refers to the classical relativistic Nambu-Goto Lagrangian and takes as additional justification the mass tower spectrum of the canonical quantized N-G Lagrangian. It possess (as does the generalized free field) much more degrees of freedom than standard QFT (but so does

in some way the infinite spin tower string as a result of the indecomposable helicity tower). However *string theoretical objects are not string-localized in any intrinsic quantum physical sense*. In fact string theorists have seen the pointlike localization in the commutator of two free string fields (this is the only case in which one has the setting of a string field theory). In order to uphold their metaphors they insist to interpret these points of causal localization as the center of mass of a string whereas the string itself does not cast a causal shadow. Tragically they fell prey to their own metaphoric language which they themselves created.

Apparently the metaphoric language is useful for the way in which they define their interaction in terms of splitting and recombining tubes. The unusual and highly suspicious aspect of string theory as compared to the Wigner classification of matter comes about by the fact that the string theory arena of the Poincaré group representation is the target space of a chiral QFT (for whatever such classical words mean in the quantum context). But no matter how it arises, modular localization, which is always intrinsically related to the representation of the Poincaré group, is the *sovereign about quantum localization* and not some classical string aspect of a N-G Lagrangian. Whereas (luckily for the development of QFT) the classical and the quantum notion coalesce in the pointlike case, this is not so for string localization. The rule is: *quantum strings cannot be obtained from quantization and classical strings do not imply quantum strings*.

There are only two types of genuine string localized objects in Minkowski spacetime: *decomposable* strings which result from pointlike fields by smearing over infinitely thin tubes, or indecomposable strings as they arise either from Wigner representation of the third kind from the vector/tensor-potentials of massless higher helicity representations.

## 2 Partial and complete invisibility as spacetime interpretation of confined and dark matter

The existence of local observables is a prerequisite for measuring properties of quantum matter. There are two notions of localization, the Born-localization of wave functions which in the relativistic context becomes frame-dependent Newton-Wigner localization and the above explained genuinely covariant modular localization<sup>13</sup>. It is only the first which comes with a (Born) probability interpretation and projection operators which are only in an macro-causal asymptotic sense (large time like separation between two such Born-localized events) consistent with a luminal-bounded propagation whereas the strictly causal modular localization has nothing to do with projectors and probabilities but rather with domains of modular involutions. In the absence of interactions B-N-W-localized states and modular localized states are, although conceptually totally different, in a fixed frame in the effective FAPP sense the same; the difference consists in an exponential tails which in case of massive matter is characterized by the Compton wave length of the particle. The idealization of a counter as a sharp "modular localizator" would lead to vacuum-polarization-caused activation in the vacuum state even if no particle is around. To avoid this zero effect we follow [19] and identify counters with members of the quasi-local observable  $C^*$ -algebra  $\mathcal{A}_{q\text{as}i}$  which is the algebra whose operators can

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<sup>13</sup>This difference in localization also leads to a significant distinction in the information theoretic entanglement of QM (Born-localization) and the KMS thermal manifestation of causally restricted states (in particular the vacuum state) whose nonobservance leads to the black hole information paradox [21].

be approximated rapidly (faster than any inverse Euclidean power) by local observables; this somewhat larger  $C^*$ -algebra contains observables which annihilate the vacuum and localize one-particle states.

The vacuum polarization at the boundary may appear as a conceptual nuisance in the measurement process, but it is of crucial importance for the understanding of astrophysical manifestations of "localization thermality" (Unruh, Hawking temperature) and the use of holographic projections onto the causal boundary for the computation of the leading  $c \ln \varepsilon$  behavior of the localization entropy in the attenuation size  $\varepsilon$  of the vacuum polarization cloud [20]. This attenuation length should not be confused with a cutoff. The latter is a forced restriction in order to obtain finite results which (if taken seriously) modifies the theory whereas attenuation of vacuum polarization is a physically well-defined method to control vacuum polarization at causal boundaries which becomes infinitely string in the limit of sharp boundaries in a given local QFT.

It is easy to see why in most work on entropy the authors insist to interpret  $\varepsilon$  as a cutoff. It originates from naively identifying a causally localized algebra in QFT with a box in QM. One of the marvelous conceptual achievements of modular theory is that it exposes a basic difference between the quantum mechanical Born localization and its relation to entanglement, and information theory and the quantum field theoretical modular localization for which the restriction of pure global states to modular localized algebras creates a completely different type of thermal entanglement which cannot be related to information theory [21]. This is a point where mathematical refinement and conceptual depth go hand in hand and I have thought a very long time before deciding to avoid the name "entanglement entropy" for that causal localization-caused (i.e. finite propagation speed) thermal entropy thus hoping to avoid at least those conceptual confusions caused by inappropriate semantics. The same setting also reveals that the so-called short energy violation of the energy conservation by vacuum fluctuations based on an interpretation of Feynman graphs belongs to the metaphoric part of particle physics. The correct statement is that a vacuum excitation  $A|0\rangle$  caused by a local observable in an interacting model has components to arbitrary high particle number i.e.  $\langle p_1, \dots, p_n | A | 0 \rangle \neq 0$  for all  $n > 1$  with matching quantum numbers. Via crossing this vacuum cloud determines all the formfactors and hence the whole theory. There is no short time violation of the energy-momentum involved here, one only needs a local vacuum excitation.

As the physical interpretation of the third kind Wigner matter was not properly understood as long as it was analyzed under the prejudice of standard QFT, the entropy related to causal or event horizons will remain a metaphoric concept as long as the entanglement in the sense of information theory is confused with the thermal manifestation by infinite vacuum polarization clouds near causal boundaries. With the continuation of such misunderstandings the future of the "black hole information paradox" is guaranteed.

The "darkness" also forces one to rethink well-known time-honored concepts as that of a thermodynamic limit, the equation of state and the definition of an energy-stress tensor. As a result of the indecomposable semiinfinite string-like nature of the spin tower representation the quantization box approximation of KMS states (which do exist [3]) cannot be done in the standard way. For the same reasons the standard methods do not work for the equation of state and the energy-stress tensor. A similar conceptual problem was recently solved in connection with the definition and calculation of the localization entropy in theories with normal quantum matter. The nature of a local algebra with respect to the vacuum is identical to that of a global algebra with respect to a KMS state which suggests

that there should be a relation between the thermal aspects caused by localization and the standard heat bath thermal setting. Using the technique of holographic projection which converts the bulk algebra into an extended chiral algebra and combining it with the inverse Unruh effect for chiral theories [18][20] one find this relation. Interestingly the logarithmic dependence on the attenuation distance  $\varepsilon$  which one has to concede to the vacuum polarization cloud at the horizon is nothing else than the result of a conformal transformation applied to a length factor in the standard heat bath volume factor whereas the two other length factors make up the transverse area factor. In this way one learns that the area law of localization entropy is a general structural property of QFT. There is hope that the new thermal problems of the third kind of matter can also be solved by similar ideas. But Wigner's problem with placing this matter into a heat bath shows that this is nontrivial.

It is evident that the semiinfinite strings of the infinite spin kind are not measurable by any counter which works as a localizer (the most basic role of a counter); unlike a composite string which results from integrating a pointlike field over an infinitely thin tube, it is simply not possible to register a finite piece of an *indecomposable semi-infinite object*, they are not members of the quasilocal algebra and they cannot be chopped up into compact pieces. In fact the above argument showed that the infinite spin string does not even contain any *composite* subobject which can be registered in a counter. This leaves of course the possibility of an indirect evidence if such strings could interact with the compactly localizable standard matter, in this case the third kind of matter has a chance of being detected with the planned underground dark matter detecting devices.

From my difficulties in formulating such an interaction I tend to believe that infinite spin matter is non-gravitationally inert (but I do not have a proof for this, further research is necessary). So a total inertness in those planned DM laboratory experiments would eliminate all other proposals (WIMPs,..) as DM candidates in my view would favor the third kind of quantum matter over all other proposals. The arguments of cosmologist/astrophysicists leading to lower mass limits are only applicable to normal matter and not infinite spin matter; the latter has not been studied sufficiently and analogies to massive/massless ordinary matter are not reliable.

As we have seen in the previous section the change from Wigner's first kind of massive matter to the third kind of massless infinite spin matter is however not quite that abrupt as it appears. It was mentioned there that the second kind of quantum matter associated to zero mass finite helicity representations does not admit certain generating tensor/spinor generating potential fields. The best known case is that of the vector potential for the photon representation. As already verbally stated there (7), as a consequence of the Hilbert space positivity requirements there exists no covariant pointlike vector potential  $A_\mu(x)$  with the photon generating property but there is a semiinfinite string-localized covariant generating field in the Wigner-Fock space

$$\exists A_\mu(x, e) \text{ such that } \langle k, h = \pm 1 | A_\mu(x, e) | 0 \rangle \neq 0 \quad (11)$$

Such stringlike "potentials" exist for all helicities  $h \geq 1$  "field strength". Besides the improved short distance properties and the increase of the realm of perturbative renormalizable interactions one may ask: is there an intrinsic representation theoretic reason for introducing interaction-free potentials on top of the field strength which already generate the system of local observables? As stated after (7) such generating tensor/spinor potentials exist, so that as in the massive case a given Wigner spin/helicity  $h$  can be described

by the full range of (if necessary string-localized) tensor/spinor fields of tensor/spinorial degree which is only subject to the inequality in (7). In this case it turns out that the previously mentioned Haag duality (6) has an interesting multi-connected generalization which signals the presence of semiinfinite string-like vector-potential or higher potential. As an illustration we mention the photon representation for which the doubly connected Minkowski spacetime region of a toroidal diamond  $\mathcal{T}$  (the causal completion of a 3-dim. torus) violates Haag duality [15] thus leading to a genuine inclusion

$$K(\mathcal{T}) \subsetneq K(\mathcal{T}')' \quad (12)$$

$$\mathcal{A}(\mathcal{T}) \subsetneq \mathcal{A}(\mathcal{T}')' \quad (13)$$

where in the second line we wrote the associated algebra inclusion which results from the application of the Weyl functor from the spatial inclusion of  $K$  subspaces of the Wigner representation space.

It is easy to see that the existence of string-localized vector potentials  $A_\mu(x, e)$  leads to a global element which goes once around in  $\mathcal{T}$  and cannot be obtained by patching together local pieces i.e. is not taken care of by the additivity property within a doubly connected spacetime region which was used in the definition of the left hand side. This inclusion relation is totally intrinsic, i. e. it is not an aspect of a particular wave function but rather of the massless finite helicity matter representation itself. Intrinsic localization properties can however be expressed in terms of singular generators. It can be shown that besides point- and semiinfinite string-like generators there is no need to introduce generators on surfaces and higher dimensional submanifolds in order to generate the whole net of algebras. In this sense QFT is much more economical than classical field theory where there are no such generating objects. The reader may have noticed in the course of reading this article that the spirit of local quantum physics (LQP) also referred to as algebraic QFT (AQFT) consists to interpret all physical properties of a system to intrinsic properties of localized subsystem thus avoiding to touch individual operators belonging to the subsystem. Thus the properties as the classical gauge principle which selects among all possible covariant classical couplings those which comply with the Maxwell theory are on the quantum level replaced by the requirement of finding in a (necessary stringlike) vector potential setting sufficiently big compactly localizable subalgebras.

Traditionally such problems as the nonexistence of photon-generating covariant vector potentials have been treated by enforcing pointlike covariant potentials through circumventing the above no-go theorem with the help of an indefinite metric extension of the Hilbert space. The BRST cohomological structure secures the correct (perturbative) description of the BRST-invariant observables (which correspond to the classical gauge invariants). But the conceptual prize to pay for saving the pointlike Lagrangian perturbation formalism is the mystification by trading the non-observable strings with a pointlike BRST ghost formalism. The BRST also weakens the mathematical aspects of the formalism since one loses the powerful Hilbert space techniques (inequalities). Working with the string localized potentials there is no cohomological descend and the original Wigner-Fock structure of the Hilbert space is maintained throughout, just as in  $s < 1$  models.

One certainly would expect a better description of such old incompletely solved problems as the *infrared aspects in QED*; in fact the string directions  $e$  in string-photon propagators are natural infrared parameters and all processes with incoming and outgoing charge lines depend on them; they tend to delocalize the charges. Only charge neutral

processes as elastic scattering of two photons involve strictly localizable infrared-finite objects. One may speculate that in case of several mutually couples vector potentials there will be compactly localizable subalgebras (observable gluonium) generated by pointlike covariant observable composite fields. However the vector gluons as zero mass objects are string localized and hence not observable themselves That certain indecomposable objects as vector gluons (which in the presence of interactions can only be obtained via asymptotic scattering theory) remain invisible while there are still *composite observable objects* is of course something one has gotten use to under the euphemism "confinement", whereas its radical extension namely *complete invisibility* (the darkness of the astrophysicists) is certainly unexpected and would be considered pure science fiction if there would not be the time-honored third family in Wigner's positive energy representation list which via its semiinfinite string like localization properties precisely shows this behavior. To view the confinement problems of gauge theory as a kind of pre-stage of complete invisibility and inertness should also have a backreaction in the sense of constructing a radical re-formulation of gauge theory for which confinement is explained in terms localization-caused partial darkness.

Although there is no reason to introduce semiinfinite stringlike localized objects for the massive representation family since the full range of objects of the spinorial calculus (7) are available, this situation changes in the presence of interactions. It is well known that pointlike localized massive vector fields have short distance dimension two instead of one as in the scalar case. Hence there is no pointlike vector potential which could lead to a perturbative renormalizable coupling. However there plenty of covariant stringlike generators  $A_\mu(x, e)$  with short distance dimension one which lead to renormalizable couplings. Traditionally renormalizable interactions have been formulated in the setting of (massless) gauge potentials using the somewhat metaphoric picture of a vectormeson receiving its mass via the Schwinger-Higgs screening mechanism. Replacing at least some metaphoric aspect by a more intrinsic setting one may (still within a BRST ghost setting) start with vectormesons which are already massive and convince oneself that the consistency of the BRST formalism requires the presence of a scalar particle (a "Higgs", but now a normal scalar particle without the Higgs condensate) [22]. As stated before, the ultimate step away from metaphors towards intrinsicness would consist in removing the ghosts altogether and work with string localized massive vector potentials instead. Only in this ghostfree setting the question of whether locality requires that selfinteracting massive vectormesons always accompanied by a "scalar satellite"<sup>14</sup> or if there are also "Higgsless" selfinteracting massive vectormesons. In the present setting the question of whether the Higgs mechanism is necessary for self-interacting massive vectormesons boils down to the statement that the consistency of the BRST ghost formalism requires the presence of a scalar satellite particle, but this is not the physical answer one is looking for. A perturbation theory involving string-like localized covariant fields is not easy and has not been done yet, partially because it involves new conceptual problems in adjusting the Epstein-Glaser iteration arguments [23].

Since any substance which carries energy cannot hide from the influence of gravitation there is a deep paradigmatic problem here: how does gravity interact with a substance which is presumably totally inert relative to any normal (compactly localizable) matter? Since the infinite spin matter has no classical Lagrangian of which it can be considered to

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<sup>14</sup>By starting with massive vectormesons there is no "vacuum condensate" which is the hall-mark of the Higgs mechanism.

arise by quantization, it is tempting to think that the understanding of quantum gravity is inexorably linked to that of semiinfinite string-localized third kind of positive energy matter.

Needless to add the present author shares the widespread opinion that dark energy has nothing to do with an unknown quantum substance beyond the normal and dark matter. The most plausible explanation is that the cosmological state in the presence of curvature cannot be the vacuum whose existence and well-known properties are inexorably tied to Poincaré symmetry. Beyond Poincaré symmetry the quantum principle of local covariance takes over and relates geometrical properties with those of operators and states of quantum matter. The naive expectation would be that for small curvature the spacetime dependent energy density is close but not equal to zero. Unfortunately there are (depending on the kind of matter) new couplings of quantum matter (no cutoffs please!) to the curvature tensor which in the logic of renormalization theory have to be treated as new unknown parameters and for whose determination one needs several observations about different aspects of the cosmological state than just its energy density. But there is no problem in principle [21] to compute the energy density minimized over an infinite subset of states<sup>15</sup> [24] for which one takes typically (for free field) quasifree Hadamard states [25]. The great popularity which the DE issue enjoys presently over all other problems (including DM) belongs to those fashions which are hard to justify in scientific terms. There is nothing more supportive for a fashion than creating its own problems or paradoxes.

In concluding I would like to add that the motor behind this investigations was not only their conceptual appeal but also the historical charm resulting from the possibility that the discoverer of the DM Fritz Zwicky and his contemporary, the protagonist of particle classification theory Eugene Wigner, may have more in common than anybody would expect. As a theoretical physicist interested in conceptual problems I always admired Wigner's strict insistence in exploring known principles before doing mind games. Whereas the traditional way of valuating observations essentially did not change from the time of Zwicky, the same cannot be said about modern particle theory where the number of researchers following the intrinsic logic of theoretical principles ala Wigner has gone down in favor of mind games.

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<sup>15</sup>In a generic Lorentzian manifold there seems to be no way to distinguish a particular state.

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